Relativity appendix 2: general relativity

Roughly speaking, general relativity extends the picture of spacetime in special relativity in the same way that the geometry of curved surfaces generalizes the geometry of flat surfaces. The key point is that any curved surface looks flat when you're close to it. Earth is a sphere, but this had to be discovered, because the curvature is observable only on a scale of several kilometers. Up close, the surface of Earth looks flat.

When we say the surface of Earth is spherical, we state only an approximate correspondence. We mean that to a certain degree of accuracy, distances between features on Earth correspond to distances that would be measured on an idealized sphere. When you look more closely, you see that Earth's surface has mountains and valleys—and over millions of years these surface features change. But if we take Earth's surface at a given moment, there will be some mathematical two-dimensional surface it corresponds to. Of course, if we look too closely, on the atomic scale, we can't distinguish exactly where the surface is, so to call it two-dimensional is again an idealization. But as long as our measurements are coarse enough, we can establish a correspondence between the surface of Earth and some particular mathematical surface.

Similarly, the association of Minkwoski spacetime with the geometry of the world is only an idealization. When we look sufficiently closely, we can see that there are analogues of mountains and valleys in the geometry of spacetime. These are gravity.

The connection between gravity and geometry is the achievement of Einstein's theory of general relativity, which he invented in a laborious process from 1907 or so to 1915.[1] The claim that gravity is geometry is stunning and represents another step forward in the mathematicization of our conception of nature. But before giving in to the undeniable charms of this idea, let's understand what it really means. How could gravity correspond to geometry?

General relativity is a dynamic theory of spacetime, with special relativity at its core—in the sense that in general relativity you can find observers for whom special relativity seems approximately true, as long as they observe only for short times and over a limited area. Gravity is the phenomenon that dictates who these special observers are.

General relativity—like its older sister, special relativity—has an operational face and a mathematical face. To understand what the theory asserts about nature, it's best to start with the first and introduce the second gradually.

Let's go back to the three-year-old's question: Why do things fall? What is pulling on Danny's ball, or a child's toy, when it falls to Earth?

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[1] Einstein, Albert, "The Foundation of the General Theory of Relativity," *Annalen der Physik,* 49(7): 769–822 (1916).

A big clue is Galileo's discovery that everything falls with a constant acceleration. What did Newton do with this clue? Newton posited a universal force that pulls on every object proportionally to its mass. But "mass" is defined, more fundamentally, as the ratio of force to acceleration (*F=ma*), so the more massive a body, the less acceleration it receives from a given force. If the force increases proportionally to the mass but the acceleration decreases proportionally to that mass, then the acceleration with which a body falls doesn't depend on its mass. Nor does it appear to depend on anything else.

This, however, makes the fact that all bodies fall with the same acceleration into a kind of coincidence. But Einstein didn't believe in coincidences-at least not in the laws of nature. If all bodies fall the same way despite their mass, this must be a principle of nature. Like all such principles, it must not come about as the result of adjusting two quantities so that they're equal. The very fact that this adjustment had to be made in Newton's theory was, for Einstein, a clue that there must be a deeper understanding that would allow no choice about the fact that all bodies fall the same way.

In Newton's physics, there's no need to explain why something is moving if it's moving with a constant speed and direction. This is a consequence of the relativity of inertial motion. What does require explanation is any deviation in either the speed or direction of motion, which is acceleration. Newton explains acceleration as due to forces, which are interactions with other bodies. Falling bodies accelerate, so that requires an explanation, so there must be a gravitational force.

Einstein turned this around and posited that falling is the natural state. Everything falls the same way because falling is the natural motion. Einstein elevated this to a principle, which he called the *equivalence principle*. This is the most intuitive of ideas about nature. Imagine that you're in an elevator (in a very long elevator shaft) whose cable has been cut, so that the elevator and everything in it is falling. Einstein's equivalence principle states that there's no way for you, inside the elevator, to tell that you're falling —or, indeed, moving at all. Einstein asks you to compare what you would experience in a free-falling elevator with what you would experience if the elevator were floating freely in space. His principle says that there's no way you could distinguish these two situations without looking outside.

Falling is not what requires explanation, because it's natural. But standing (say) does. If standing is unnatural, it requires a force, and indeed we do feel a force when we are standing—the force pulling us to the ground. Einstein's insight is as deep as it is ordinary: The only time we don't feel a force is when we're falling. *We feel gravity because we are not falling*. What we feel is not the force of gravity, it is the force of the floor or the chair pushing up on us, keeping us from falling.

Thus, a free-falling person observes the world around her to be described by special relativity as long as she observes only for small intervals of time and small regions of space. This is the basis of the analogy between general relativity and curved geometry.

The second part of the equivalence principle involves situations in which we are not falling. We feel a force pushing us toward the floor. How do we know this is gravity? Couldn't it be the result of our room accelerating upward? Einstein asks us to compare our situation in a room on Earth with that of the passengers in a rocket ship accelerating into space. They feel pushed toward the floor by the acceleration of their rocket ship. We feel pushed toward the floor because the floor is accelerating us compared to free-falling observers.

Figure 16: [[room and rocket ship]]

Einstein postulated that these two situations are also indistinguishable. So, as you're sitting in your room or in a café reading this, you can't tell, without looking outside, whether you're in a café on Earth or a café in a spaceship accelerating upward.[2]

Einstein had this wonderful idea around 1907. One of its first consequences is his deduction that the path of light is bent by a gravitational field. The reason is simple. The passengers in an accelerating rocket ship would see a beam of light that was moving perpendicular to their motion to be accelerated toward the floor of the ship. Nothing is happening to the beam of light; it is just that as it crosses the spaceship, the floor accelerates upward toward it.

According to Einstein's principle of equivalence, we should see all such phenomena exactly the same in a room on Earth. So if we shine a laser beam across our room we should see it bend towards the floor as it travels, because each photon as it moves is being accelerated towards the floor.

This is a simple argument, but the conclusion that light falls in a gravitational field, so that its path bends, was confirmed by Arthur Eddington's observation of shifted starlight during a solar eclipse in 1919. The effect has by now been so well confirmed that it serves as a tool to detect, by its distortion of the image of an astronomical object, the presence of intervening dark matter.

But photons were previously thought to travel in a straight line. What are we to make of the fact that their paths are now curved by the presence of matter? The most direct interpretation is that the geometry of spacetime is no longer that given by Minkowski but a curved geometry. There are no straight lines on a sphere, but there is a concept that corresponds to that. A property of a straight line on a plane is that it is the shortest distance between two points—a path called a geodesic. This concept can apply to a curved surface. On a flat surface, the geodesics are straight lines, but on a curved surface, on which no lines are straight in the usual sense, there are also geodesics**.**

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[2] This is the origin of the science-fiction-staple set, which is the spaceship in which gravity is created by rotation. You may not want to accelerate all the way to Jupiter, but you can rotate your spaceship so as to produce "artificial gravity."

This is familiar to air travelers, who fly the shortest distance between two cities; on a flat map, their route would appear curved. Nor does the surface need to be that of a sphere; no matter how complicated a curved surface is, you can still define geodesics.

Figure 18: [[geodesics]]

Einstein proposed that the geometry of spacetime is defined by the motion of matter. Rather than specifying the geometry of spacetime once and for all, he regarded it as something determined dynamically. Furthermore, like all dynamic phenomena, it had to be established by observation. To do this, he specified that free-falling objects move along geodesics in spacetime, as do the paths of photons.

Gravity is now manifested by the curvature of spacetime. Objects freely fall toward Earth because they are following the geodesics of the spacetime geometry.
Note that what is curved here is not the geometry of space but the geometry of spacetime. So general relativity invites us to think about the world as a spacetime, rather than as spaces evolving in time. Thus it confirms the utility of the block-universe picture and by so doing contributes to the expulsion of time from physics.

Note also that mathematical notions have become essential to the physical explanation here. The notion of a geodesic is a mathematical idea and an irreducible part of the explanation for gravity in general relativity—although this is not as radical a shift as it may seem. The notion of a straight line is a mathematical idea too, and it played a no less essential role in Newton's explanation of motion.

But what determines what the spacetime geometry is? Einstein proposed a set of laws to describe how matter affects it, so spacetime is no longer to be specified *a priori*. It is now a solution to a dynamic equation. This partly undoes the move that Minkowski made when he declared that his absolute geometry replaced absolute time as a fixed structure within which motion is defined. The geometry of spacetime is now full of valleys, which is where matter most strongly curves it. The geometry is also decorated by waves**,** which travel through it carrying energy, very much like water waves fluttering the surface of a pond.